Tachyon Vacuum Solution in Open String Field Theory with Constant B Field

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Abstract

We show that Schnabl's tachyon vacuum solution is an exact solution of the equation of motion of Witten's open bosonic string field theory in the background of constant antisymmetric two-form field. The action computed at the vacuum solution is given by the Dirac-Born-Infeld factor multiplied to that without the antisymmetric tensor field.

1 Introduction

Recently there has been much interest in Witten's open string field theory (OSFT) for bosonic strings [1]. It is a natural framework to discuss the nonperturbative physics of D-branes as demonstrated in [2]. Several conjectures were made on tachyon condensation in [3], and the open-string tachyon was identified with the unstable mode of the D-brane. The first conjecture states that the tension of the D-brane is given by the height of the tachyon potential from the tachyon vacuum. In OSFT, Schnabl proved it by constructing an analytic tachyon vacuum solution [4]. Since then, Schnabl's method [5, 6] has been further developed to prove Sen's the third conjecture [7] as well as to study related problems analytically in OSFT, including rolling tachyon [8–20].

In this paper we would like to consider Witten's OSFT in the background of constant antisymmetric two-form field B and find the tachyon vacuum solution. This background has attracted attention since, in the presence of the B field, the end points of the open strings become noncommutative and low energy dynamics of the D-branes is described either by commutative or noncommutative gauge theories [21]. Witten's OSFT in the constant B field has been studied in [22–24], and it is verified that the action has the same form as the corresponding action on trivial background but it has Moyal type noncommutativity in addition to the ordinary Witten's star product and string coupling constant [22, 23]. We observe that Schnabl's tachyon vacuum solution [4] still satisfies the OSFT equation of motion even in this background. By direct substitution of the solution into the action we identify the tension of D25-brane in the presence of constant B field, which is nothing but the D25-brane tension multiplied by the Dirac-Born-Infeld (DBI) factor $\sqrt{-\det(\eta + 2\pi\alpha' B)}$. Since B field appears only through the gauge invariant combination $B + F$ where F is the field strength on the D-brane [21], this result is consistent with the DBI action obtained by summing over all disk diagrams with external constant F.

2 Constant B Field and Tachyon Vacuum

Dynamics of bosonic strings including off-shell contributions is described by Witten's OSFT [1], in which the action is

$$S(\Phi) = -\frac{1}{\alpha'^3 g_\mathrm{o}^2} \int \left(\frac{1}{2}\Phi * Q_\mathrm{B}\Phi + \frac{1}{3}\Phi * \Phi * \Phi \right) \tag{2.1}$$

$$= -\frac{1}{\alpha'^3 g_\mathrm{o}^2} \left(\frac{1}{2}\, {}_{12}\langle V_2||\Phi\rangle_1 Q_\mathrm{B}|\Phi\rangle_2 + \frac{1}{3}\, {}_{123}\langle V_3||\Phi\rangle_1 \Phi\rangle_2 \Phi\rangle_3 \right), \tag{2.2}$$

where g_o, Φ, $*$, and Q_B in the first line (2.1) are the open string coupling constant, a string field, Witten's star product, and the BRST charge, respectively. In the second line, ${}_{12\ldots n}\langle V_n|$ is the n-string overlap vertex and $|\Phi\rangle_n$ is the n-th string state. From (2.2) the equation of motion is

$$ {}_{12}\langle V_2|Q_\mathrm{B}|\Phi\rangle_2 + {}_{123}\langle V_3||\Phi\rangle_2|\Phi\rangle_3 = 0. \tag{2.3}$$

The analytic tachyon vacuum solution is known to be [4]

$$|\Psi\rangle = \lim_{N\to\infty}\left(\sum_{n=0}^{N}\psi'_n - \psi_N\right), \tag{2.4}$$

where ψ_n is expressed in terms of ghosts and wedge state $|n\rangle$, $(n = 0, 1, \ldots)$,

$$\psi_n = \frac{2}{\pi}c_1|0\rangle * |n\rangle * B_1^L c_1|0\rangle, \tag{2.5}$$

and $\psi'_n \equiv \frac{d\psi_n}{dn}$. B_1^L is represented in terms of b ghost as

$$B_1^L = \int_{C_L}\frac{d\xi}{2\pi i}(1+\xi^2)b(\xi), \tag{2.6}$$

where the contour C_L runs counterclockwise along the unit circle with $\mathrm{Re}(\xi) < 0$. In (2.4), the coefficient of the so-called phantom piece ψ_N has to be -1 for the solution to satisfy the equation of motion when contracted with itself [5,6],

$${}_{12}\langle V_2||\Psi\rangle_1 Q_{\mathrm{B}}|\Psi\rangle_2 + {}_{123}\langle V_3||\Psi\rangle_1|\Psi\rangle_2|\Psi\rangle_3 = 0. \tag{2.7}$$

Evaluation of the action for the Schnabl's solution gives

$$S(\Psi) = \frac{1}{6\alpha'^3 g_{\mathrm{o}}^2}\,{}_{123}\langle V_3||\Psi\rangle_1|\Psi\rangle_2|\Psi\rangle_3 = \frac{\mathrm{Vol}_{26}}{2\pi^2\alpha'^3 g_{\mathrm{o}}^2} \equiv \mathcal{T}_{25}\mathrm{Vol}_{26}, \tag{2.8}$$

where $\mathrm{Vol}_{26} = \int d^{26}x$ represents the spacetime volume factor and $\mathcal{T}_{25}$ is the tension of 25-dimensional space-filling brane. Apart from the volume factor, this coincides with the tension of D25-brane, in consistent with Sen's conjecture [3].

In the presence of a constant antisymmetric two-form field, the string worldsheet action is written as

$$\frac{1}{2\pi\alpha'}\int d^2z\,(g_{MN} - 2\pi\alpha' B_{MN})\partial X^M\bar{\partial}X^N + S_{\mathrm{gh}}, \tag{2.9}$$

where S_{gh} is the b, c ghost contribution. We denote the Greek indices $\mu,\ \nu$ as the directions that the antisymmetric tensor field is nonzero, $B_{\mu\nu} \neq 0$. The classical solution of the equation of motion under the boundary condition $E_{\mu\nu}\partial_{\bar{z}}X^\nu = (E^T)_{\mu\nu}\partial_z X^\nu$ with $E_{\mu\nu} = g_{\mu\nu} + 2\pi\alpha' B_{\mu\nu}$ is given by

$$\begin{aligned} X^\mu(z,\bar{z}) &= \tilde{x}^\mu - i\alpha'\left[(E^{-1})^{\mu\nu}\ln\bar{z} + (E^{-1T})^{\mu\nu}\ln z\right]p_\nu \\ &\quad + i\sqrt{\frac{\alpha'}{2}}\sum_{n\neq 0}\frac{1}{n}\left[(E^{-1})^{\mu\nu}\alpha_{n,\nu}\bar{z}^{-n} + (E^{-1T})^{\mu\nu}\alpha_{n,\nu}z^{-n}\right], \end{aligned} \tag{2.10}$$

where $\tilde{x}^\mu$'s are noncommutative coordinates satisfying

$$[\tilde{x}^\mu,\, \tilde{x}^\nu] = i\theta^{\mu\nu}, \qquad \theta^{\mu\nu} = -(2\pi\alpha')^2(E^{-1}BE^{-1T})^{\mu\nu}. \tag{2.11}$$

The commutation relations among operators are

$$[\alpha_{n,\nu},\, \alpha_{m,\nu}] = nG_{\mu\nu}\delta_{n+m,0}, \quad [x^\mu,\, p_\nu] = i\delta^\mu_\nu, \tag{2.12}$$

where $G_{\mu\nu}$ is the inverse matrix of $G^{\mu\nu} = (E^{-1}gE^{-1T})^{\mu\nu}$ and $x^\mu = \tilde{x}^\mu + \frac{1}{2}\theta^{\mu\nu}p_\nu$ is the center of mass coordinate.

The corresponding OSFT action in the constant B field background is [22, 23]

$$\begin{aligned} S_B &= -\frac{1}{\alpha'^3 G_{\rm o}^2}\int\left(\frac{1}{2}\,\Phi\star\tilde{Q}_{\rm B}\Phi + \frac{1}{3}\Phi\star\Phi\star\Phi\right) \\ &= -\frac{1}{\alpha'^3 G_{\rm o}^2}\left(\frac{1}{2}\ {}_{12}\langle\hat{V}_2||\Phi\rangle_1\tilde{Q}_{\rm B}|\Phi\rangle_2 + \frac{1}{3}\ {}_{123}\langle\hat{V}_3||\Phi\rangle_1|\Phi\rangle_2|\Phi\rangle_3\right), \end{aligned} \tag{2.13}$$

where $G_{\rm o}$ is the open string coupling in the presence of constant B field [21],

$$G_{\rm o}^2 = g_{\rm o}^2\sqrt{\frac{\det(g+2\pi\alpha' B)}{\det g}}\,. \tag{2.14}$$

In $S_{\rm B}$, $\star$ in the first line is Witten's star product modified by the Moyal-type noncommutativity and ${}_{12\ldots n}\langle\hat{V}_n|$ in the second line is the n-string overlap vertex corresponding to the product $\star$ in the B field background. The BRST charge $\tilde{Q}_{\rm B}$ is not affected by the B field except that the metric in $\tilde{Q}_{\rm B}$ changes to the open string metric $G_{\mu\nu}$. The equation of motion of (2.13) is

$${}_{12}\langle\hat{V}_2|\tilde{Q}_{\rm B}|\Phi\rangle_2 + {}_{123}\langle\hat{V}_3||\Phi\rangle_2|\Phi\rangle_3 = 0, \tag{2.15}$$

where the mode expansion of ${}_{12}\langle\hat{V}_2|$, $\tilde{Q}_{\rm B}$, and ${}_{123}\langle\hat{V}_3|$ are expressed in terms of $G_{\mu\nu}$, $\alpha_{n,\mu}$, p_μ, x^μ, b_n, and c_n defined above.

Now, we find a nontrivial homogeneous solution of this equation which represents the tachyon vacuum. Let us first introduce new modes [22]

$$\hat{\alpha}^\mu_n = \left(E^{-1T}\right)^{\mu\nu}\alpha_{n,\nu}, \quad \hat{p}^\mu = \left(E^{-1T}\right)^{\mu\nu}p_\nu, \quad \hat{x}_\mu = E_{\mu\nu}x^\nu. \tag{2.16}$$

Then the commutation relations become those without B field,

$$[\hat{\alpha}^\mu_n,\, \hat{\alpha}^\nu_m] = ng^{\mu\nu}\delta_{n+m,0}, \quad [\hat{x}_\nu,\, \hat{p}^\mu] = i\delta^\mu_\nu, \tag{2.17}$$

and the bilinear form of α^μ_n with metric $G_{\mu\nu}$ is replaced by that of the transformed modes $\hat{\alpha}^\mu_n$ with metric $g_{\mu\nu}$, i.e.,

$$G^{\mu\nu}\alpha_{n,\mu}\alpha_{m,\nu} = g_{\mu\nu}\hat{\alpha}^\mu_n\hat{\alpha}^\nu_m, \tag{2.18}$$

where $\alpha_{0,\mu} = \sqrt{2\alpha'}\, p_\mu$. Then, ${}_{12}\langle \hat{V}_2|$ and ${}_{123}\langle \hat{V}_3|$ are rewritten in terms of the new modes, $g_{\mu\nu}$, $\hat{\alpha}^\mu_n$, $\hat{p}^\mu$, $\hat{x}_\mu$, b_n, and c_n, and are related with those in the absence of B field as [22, 23],

$$ {}_{12}\langle \hat{V}_2| = {}_{12}\langle V_2|, \qquad {}_{123}\langle \hat{V}_3| = {}_{123}\langle V_3| \exp\left(-\frac{i}{2} \sum_{r<s} \theta^{\mu\nu} p^{(r)}_\mu p^{(s)}_\nu \right). \tag{2.19} $$

Also, $\tilde{Q}_{\rm B}$ has the same form as $Q_{\rm B}$ with the oscillators replaced by the new ones.

With these forms of overlap vertices and the BRST charge, it is not difficult to see that the Schnabl's tachyon vacuum solution (2.4) is still the solution of the equation of motion (2.15) even in the presence of constant B field. For this, we need to show that (2.15) holds when contracted with any state in the Fock space as well as with the solution itself [5,6]. First note that the kinetic term reduces to that in the absence of the B field due to (2.19) and (2.17). Furthermore, since the tachyon vacuum solution is a homogeneous solution [4], we have $p^{(r)}_\mu |\Psi\rangle_r = 0$ and hence the Moyal type noncommutativity disappears in the cubic term. Then, the resulting equation coincides with the equation without the background B field (2.7) for homogeneous string fields. This is true for the contraction with either any state in the Fock space or the string field itself. This completes the proof.

From now on let us discuss physical properties of the solution. In the absence of the B field, the calculation of the action (2.1) for the Schnabl's analytic vacuum solution gives the exact value of the tension $\mathcal{T}_{25}$ of 25-dimensional space-filling brane [4] as in (2.8). Since all the B field dependence in the action (2.13) appears only through the open string coupling constant G_o and the transformed center of mass coordinate $\hat{x}^\mu$, the value of the action (2.13) at the tachyon vacuum solution with constant B field gives

$$ \begin{aligned} S_B(\Phi = \Psi) =& \frac{1}{6\alpha'^3 G_o^2} {}_{123}\langle V_3||\Psi\rangle_1 |\Psi\rangle_2 |\Psi\rangle_3 = \frac{1}{2\pi^2 \alpha'^3 G_o^2} \int d^{26}\hat{x} \\ =& \frac{\mathrm{Vol}_{26}}{2\pi^2\alpha'^3 G_o^2} |\det(\eta + 2\pi\alpha' B)| = \mathcal{T}_{25} \mathrm{Vol}_{26} \sqrt{-\det(\eta + 2\pi\alpha' B)}\,, \end{aligned} \tag{2.20} $$

where $\eta_{\mu\nu}$ is the flat spacetime metric, $|\det(\eta + 2\pi\alpha' B)|$ is the Jacobian factor for the coordinate transformation given in (2.16), and we used the relation $G_o^2 = \sqrt{-\det(\eta + 2\pi\alpha' B)}\, g_o^2$ from (2.14). Since we are considering the case of a constant B field, the result (2.20) is consistent with the DBI action obtained by summing over all the disk diagrams with external gauge field legs of constant electromagnetic field strength $F_{\mu\nu}$ as boundary terms [25], considering the fact that B field appears only through the gauge invariant combination $B_{\mu\nu} + F_{\mu\nu}$. The result (2.20) is natural because the Schnabl's tachyon vacuum solution in the presence of constant B field background is an exact analytic solution of classical SFT equation and does not involve any off-shell contribution.

3 Conclusion

In this paper, we showed that Schnabl's tachyon vacuum solution is an exact solution of the string field equation in the presence of constant B field. Since the effect of the constant B field appears only in the open string coupling and the Jacobian factor of spacetime integration, the value of the SFT action is easily computed and gives the DBI Lagrangian density multiplied by the D25-brane tension and spacetime volume, which coincides with the result of disc amplitude computation in string theory.

The obtained result may open a new direction in OSFT including both constant B field background and marginal deformations of either time or spatial dependence. It would be intriguing if the treatment of the B field becomes applicable to the cases with time and spatial dependence, which lead to higher-derivative corrections [26]. Since fundamental strings couple minimally to the antisymmetric tensor field B in string theories, our work on the OSFT in the presence of the constant B field may contribute to understanding of fundamental strings in the context of OSFT.

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